Section-I

Q.1. Attempt any SIX of the following : [12]

(i) Write down the negation of the following statements :

(a) If monsoon is good then farmers are happy.

(b) I will have tea or coffee. (2)

Sol.: (a) Monsoon is good and farmers are not happy. *(1 mark)*

(b) I will neither have tea nor coffee. *(1 mark)*

(ii) If $\begin{bmatrix} x+2y & 2 \\ -1 & x-y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 5 \end{bmatrix}$ find x and y. (2)

Sol.: By equality of matrices,

$x + 2y = 4$ ....(i) and $x - y = 5$ ....(ii)

Subtract equation(ii) from (i), we get

$$\begin{array}{rl} x + 2y = & 4 \\ x - y = & 5 \\ - \quad + & - \\ \hline 3y = & -1 \end{array} \Rightarrow y = -\frac{1}{3} \quad \textit{(1 mark)}$$

$$x = 5 + \left(-\frac{1}{3}\right) \Rightarrow x = \frac{14}{3} \quad \textit{(1 mark)}$$

(iii) Find $\frac{dy}{dx}$ if $x^3 + x^2y + y^3 - 9 = 0$. (2)

Sol.: $x^3 + x^2y + y^3 - 9 = 0$

Differentiating w.r.t. x, we get

$$3x^2 + x^2\frac{dy}{dx} + y(2x) + 3y^2\frac{dy}{dx} = 0 \quad \textit{(1 mark)}$$

$$\left(x^2 + 3y^2\right)\frac{dy}{dx} = -3x^2 - 2xy$$

$$= -x(3x + 2y)$$

$$\therefore \frac{dy}{dx} = -\frac{x(3x+2y)}{x^2+3y^2} \quad \textit{(1 mark)}$$

(iv) Evaluate $\int \frac{1}{1+\cos x}dx$. (2)

Sol.: Let $I = \int \frac{1}{1+\cos x}dx = \int \frac{1}{2\cos^2\left(\frac{x}{2}\right)}dx$

$$= \frac{1}{2}\int \frac{1}{\cos^2\left(\frac{x}{2}\right)}dx = \frac{1}{2}\int \sec^2\left(\frac{x}{2}\right)dx \quad \textit{(1 mark)}$$

$$= \frac{1}{2}\left[\frac{\tan\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\right] + c = \tan\left(\frac{x}{2}\right) + c \quad \textit{(1 mark)}$$

(v) Show that the function $f(x) = 2x^3 - 12x^2 + 41x + 20$ is increasing at every real number x. (2)

Sol.: $f(x) = 2x^3 - 12x^2 + 41x + 20$

$$f'(x) = 6x^2 - 24x + 41$$

$$= 6\left(x^2 - 4x + 4\right) + 17$$

$$= 6(x-2)^2 + 17 \quad \textit{(1 mark)}$$

$$\therefore f'(x) = 6(x-2)^2 + 17 > 0$$

for all real values of x. *(1 mark)*

$\therefore f(x)$ is an increasing function for all real values of x.

(vi) Evaluate $\int_0^{\pi/4} e^{\tan x} \cdot \sec^2 x\,dx$ (2)

Sol.: Let $I = \int_0^{\pi/4} e^{\tan x} \cdot \sec^2 x\,dx$

Put $\tan x = t$, $\sec^2 x\,dx = dt$

When $x = 0$, $t = 0$; When $x = \frac{\pi}{4}$, $t = 1$

$$\therefore I = \int_0^1 e^t\,dt = \left[e^t\right]_0^1 = e^1 - e^0 \quad \textit{(1 mark)}$$

$$= e - 1 \quad \textit{(1 mark)}$$

(vii) Evaluate $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}dx$. (2)

Sol.: Let $I = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}dx$ ....(i)

By property of definite integral

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}dx$$

$$= \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}dx \quad \text{....(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\cos x}{\sin x + \cos x} + \frac{\sin x}{\cos x + \sin x}\right)dx$$

$$= \int_0^{\pi/2} \frac{\cos x + \sin x}{\sin x + \cos x}dx \quad \textit{(1 mark)}$$

$$= \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$\therefore 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \quad \textit{(1 mark)}$$

(viii) Show that the area under the curve $y = 2x + \sin x$ between $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$ is $\frac{\pi^2}{4} + 1$ square units. **(2)**

Sol.: Area $= \int_0^{\pi/2} y\,dx = \int_0^{\pi/2} (2x + \sin x)\,dx$

$$= \left\{2\left[\frac{x^2}{2}\right] - \cos x\right\}_0^{\pi/2}$$

$$= \left[x^2 - \cos x\right]_0^{\pi/2} \quad \textit{(1 mark)}$$

$$= \left[\frac{\pi^2}{4} - \cos\left(\frac{\pi}{2}\right)\right] - [0 - 1]$$

$$= \frac{\pi^2}{4} - 0 + 1 \quad \textit{(1 mark)}$$

$$= \frac{\pi^2}{4} + 1 \text{ square units.}$$

Q.2.(A) Attempt any TWO of the following : [6]

(i) Write the converse, contrapositive and inverse of the following logical statements: "if $x < y$ then $x^2 < y^2$." **(3)**

Sol.: **Converse :** If $x^2 < y^2$, then $x < y$. *(1 mark)*

Contrapositive : If $x^2 \not< y^2$ then $x \not< y$. *(1 mark)*

Inverse : If $x \not< y$, then $x^2 \not< y^2$. *(1 mark)*

(ii) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 3 & 1 \\ 2 & 2 & 2 \end{bmatrix}$, show that $(A + B)^2 = A^2 + 2AB + B^2$. **(3)**

Sol.: $(A + B)^2 = (A + B)(A + B)$

$$= A^2 + AB + BA + B^2$$

$$= A^2 + 2AB + B^2, \text{ if } AB = BA$$

Now, $AB = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 4 \\ 1 & 3 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 4+2+2 & 0+6+2 & 4+2+2 \\ 4-1+2 & 0-3+2 & 4-1+2 \\ 4+0+2 & 0+0+2 & 4+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 8 & 8 \\ 5 & -1 & 5 \\ 6 & 2 & 6 \end{bmatrix} \quad \textit{(1 mark)}$$

$$BA = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 3 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} 4+0+4 & 8+0+0 & 4+0+4 \\ 1+3+1 & 2-3+0 & 1+3+1 \\ 2+2+2 & 4-2+0 & 2+2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 8 & 8 \\ 5 & -1 & 5 \\ 6 & 2 & 6 \end{bmatrix} \quad \textit{(1 mark)}$$

$\therefore AB = BA$

$\therefore (A + B)^2 = A^2 + 2AB + B^2$ *(1 mark)*

(iii) If $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$, find $\frac{dy}{dx}$. **(3)**

Sol.: $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right] \quad \textit{(1 mark)}$$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right] = \frac{x}{2} \quad \textit{(1 mark)}$$

$\therefore y = \frac{x}{2}$

Differentiate w.r.t. x,

$$\frac{dy}{dx} = \frac{1}{2}. \quad \textit{(1 mark)}$$

Q.2.(B) Attempt any TWO of the following : [8]

(i) If 'f' is continuous on $(-\pi, \pi)$, where

$$f(x) = -2\sin x, \quad \text{for } -\pi < x \le -\frac{\pi}{2}$$
$$= \alpha\sin x + \beta, \quad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$= \cos x, \quad \text{for } \frac{\pi}{2} \le x \le \pi$$

Find α and β. **(4)**

Sol.: Given f is continuous on $(-\pi, \pi)$.

Since f is continuous at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

$$\therefore \lim_{x \to \left(-\frac{\pi}{2}\right)^-} f(x) = \lim_{x \to \left(-\frac{\pi}{2}\right)^+} f(x)$$

$$\therefore \lim_{x \to \left(-\frac{\pi}{2}\right)^-} (-2\sin x) = \lim_{x \to \left(-\frac{\pi}{2}\right)^+} (\alpha\sin x + \beta)$$

(1 mark)

$$\Rightarrow -2\sin\left(-\frac{\pi}{2}\right) = \alpha\sin\left(-\frac{\pi}{2}\right) + \beta$$

$$\Rightarrow -2(-1) = \alpha(-1) + \beta$$

$$\Rightarrow 2 = -\alpha + \beta \quad \text{.......(i)} \quad \textit{(1 mark)}$$

Also,

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} f(x) = \lim_{x \to \left(\frac{\pi}{2}\right)^+} f(x)$$

$$\therefore \lim_{x \to \left(\frac{\pi}{2}\right)^-} (\alpha \sin x + \beta) = \lim_{x \to \left(\frac{\pi}{2}\right)^+} \cos x$$

$$\Rightarrow \alpha \sin\left(\frac{\pi}{2}\right) + \beta = \cos\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \alpha(1) + \beta = 0$$

$$\Rightarrow \alpha + \beta = 0 \quad \text{....(ii)}$$ *(1 mark)*

Adding (i) and (ii), we get

$$2\beta = 2$$

$$\Rightarrow \beta = 1 \quad \text{and} \quad \alpha = -1$$

$$\therefore \alpha = -1 \quad \text{and} \quad \beta = 1.$$ *(1 mark)*

(ii) Evaluate $\int_0^4 \frac{x}{\sqrt{4-x}} dx$. **(4)**

Sol.: Let $I = \int_0^4 \frac{x}{\sqrt{4-x}} dx = \int_0^4 x(4-x)^{-1/2} dx$

We have, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$\therefore I = \int_0^4 (4-x)[4-(4-x)]^{-1/2} dx$$ *(1 mark)*

$$= \int_0^4 (4-x)x^{-1/2} dx$$

$$= \int_0^4 \left(4x^{-1/2} - x^{1/2}\right) dx$$

$$= 4\int_0^4 (x)^{-1/2} dx - \int_0^4 x^{1/2} dx$$ *(1 mark)*

$$= 4\left[\frac{x^{1/2}}{1/2}\right]_0^4 - \left[\frac{x^{3/2}}{3/2}\right]_0^4$$ *(1 mark)*

$$= 8\left[(4)^{1/2} - 0\right] - \frac{2}{3}\left[(4)^{3/2} - 0\right]$$

$$= 8\left[\left(2^2\right)^{1/2} - 0\right] - \frac{2}{3}\left[\left(2^2\right)^{3/2}\right]$$

$$= 8[2] - \frac{2}{3}[8] = 16 - \frac{16}{3}$$

$$= 16\left[1 - \frac{1}{3}\right] = \frac{32}{3}$$ *(1 mark)*

(iii) A manufacturer can sell x items at a price of ₹ $(330 - x)$ each. The cost of producing x items is $C = x^2 + 10x + 12$. Determine the number of items to be sold so that the manufacturer can make maximum profit. **(4)**

Sol.: Total Selling Price (S.P.) $= x(330 - x)$

$= 330x - x^2$ *(1 mark)*

Given : Total cost $C = x^2 + 10x + 12$

$\therefore$ Profit, P = Total S. P. − Total Cost

$$\therefore P = \left(330x - x^2\right) - \left(x^2 + 10x + 12\right)$$

$$= -2x^2 + 320x - 12$$ *(1 mark)*

$$\therefore \frac{dP}{dx} = -4x + 320$$

Profit is maximum if $\frac{dP}{dx} = 0$ and $\frac{d^2P}{dx^2} < 0$.

Consider $\frac{dP}{dx} = 0$

$$\Rightarrow -4x + 320 = 0$$

$$\Rightarrow x = \frac{320}{4} = 80$$ *(1 mark)*

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(-4x + 320) = -4 < 0$$ *(1 mark)*

$\therefore$ Profit is maximum at $x = 80$.

Q.3.(A) Attempt any TWO of the following : **[6]**

(i) Using truth tables, show that

$p \leftrightarrow q \equiv [(p \to q) \wedge (q \to p)]$ **(3)**

Sol.:

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
1	2	3	4	5	6
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Column (5) and (6) are identical.

$\therefore p \leftrightarrow q \equiv [(p \to q) \wedge (q \to p)]$

[Marking Scheme : Column (5) and (6) 1 mark each and on mark for conclusion.]

(ii) Solve the following equations using method of reduction:

$2x - y + z = 1$, $x + 2y + 3z = 8$ and $3x + y - 4z = 1$. **(3)**

Sol.: The given equations can be written as matrix equations as follows :

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

By $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -23 \end{bmatrix}$$ *(1 mark)*

By $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & 0 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -15 \\ -8 \end{bmatrix}$$

$x + 2y + 3z = 8$ ......(i)

$-5y - 5z = -15$ .....(ii)

$-8z = -8$ ....(iii) *(1 mark)*

From (iii), $z = 1$

From (ii), $-5y - 5 = -15 \Rightarrow y = 2$

From (i), $x + 2\times 2 + 3 = 8 \Rightarrow x = 1$

$\therefore x = 1, \quad y = 2, \quad z = 1.$ *(1 mark)*

(iii) If $x^2y^3 = (x+y)^5$, find $\frac{dy}{dx}$. **(3)**

Sol.: $x^2y^3 = (x+y)^5$

Taking log on both sides,

$$\log\left(x^2y^3\right) = \log(x+y)^5$$

$$\Rightarrow 2\log x + 3\log y = 5\log(x+y)$$

Differentiate w.r.t. x,

$$\frac{2}{x} + \frac{3}{y}\frac{dy}{dx} = \frac{5}{x+y}\left[1 + \frac{dy}{dx}\right]$$ *(1 mark)*

$$\Rightarrow \left(\frac{3}{y} - \frac{5}{x+y}\right)\frac{dy}{dx} = \frac{5}{x+y} - \frac{2}{x}$$

$$\Rightarrow \left[\frac{3x+3y-5y}{y(x+y)}\right]\frac{dy}{dx} = \frac{5x-2x-2y}{x(x+y)}$$ *(1 mark)*

$$\Rightarrow \frac{3x-2y}{y(x+y)}\frac{dy}{dx} = \frac{3x-2y}{x(x+y)}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$ *(1 mark)*

Q.3.(B) Attempt any TWO of the following : **[8]**

(i) Find maximum and minimum values of $2x^3 - 15x^2 + 36x + 10$. **(4)**

Sol.: Let $f(x) = 2x^3 - 15x^2 + 36x + 10$

$$f'(x) = 6x^2 - 30x + 36$$

$$f''(x) = 12x - 30$$

$$= 6(2x - 5)$$ *(1 mark)*

For maximum and minimum, $f'(x) = 0$

$$\Rightarrow 6\left(x^2 - 5x + 6\right) = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-3)(x-2) = 0$$

$$\Rightarrow x = 3, \quad x = 2$$ *(1 mark)*

Case 1 :

When $x = 2$, $f''(2) = 6(2\times 2 - 5) = -6 < 0$

$\therefore f(x)$ is maximum at $x = 2$.

Case 2 :

When $x = 3$, $f''(3) = 6(2\times 3 - 5) = 6 > 0$

$\therefore f(x)$ is minimum at $x = 3$.

Maximum value of $f(x)$ is

$$f(2) = 2\left(2^3\right) - 15\left(2^2\right) + 36(2) + 10$$

$$= 2(8) - 15(4) + 72 + 10$$

$$= 16 - 60 + 72 + 10$$

$$= 38$$

$\therefore$ Maximum value of $f(x) = 38$ *(1 mark)*

Minimum value of $f(x)$ is

$$f(3) = 2\left(3^3\right) - 15\left(3^2\right) + 36(3) + 10$$

$$= 2\times 27 - 15\times 9 + 36\times 3 + 10$$

$$= 37$$

$\therefore$ Minimum value of $f(x) = 37$ *(1 mark)*

(ii) If $f(x) = \frac{\log(4+x) - \log(4-x)}{\sin x}$, $x \neq 0$

$= \frac{1}{2}$, when $x = 0$

is continuous at $x = 0$, find $f(0)$. **(4)**

Sol.: Since f is continuous at $x = 0$,

$$\lim_{x\to 0} f(x) = f(0)$$

$$\therefore f(0) = \lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0}\frac{\log(4+x) - \log(4-x)}{\sin x}$$ *(1 mark)*

$$= \lim_{x\to 0}\frac{\log\left(\frac{4+x}{4-x}\right)}{\sin x}$$

Dividing Nr. and Dr. by x $(x \to 0, \therefore x \neq 0)$

$$\therefore f(0) = \lim_{x\to 0}\frac{\frac{1}{x}\log\left(\frac{4+x}{4-x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\lim_{x\to 0}\log\left(\frac{4+x}{4-x}\right)^{1/x}}{\lim_{x\to 0}\frac{\sin x}{x}}, \text{ but } \lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\therefore f(0) = \frac{\lim_{x\to 0}\log\left(\frac{1+\frac{x}{4}}{1-\frac{x}{4}}\right)^{1/x}}{1}$$ *(1 mark)*

$$= \log\left[\frac{\lim\limits_{x\to 0}\left(1+\frac{x}{4}\right)^{1/x}}{\lim\limits_{x\to 0}\left(1-\frac{x}{4}\right)^{1/x}}\right]$$

$$= \log\left[\frac{\lim\limits_{x\to 0}\left\{\left(1+\frac{x}{4}\right)^{4/x}\right\}^{1/4}}{\lim\limits_{x\to 0}\left\{\left(1-\frac{x}{4}\right)^{-4/x}\right\}^{-1/4}}\right]$$

$$= \log\left[\frac{e^{1/4}}{e^{-1/4}}\right]$$ *(1 mark)*

$$\therefore f(0) = \log\left[e^{\frac{1}{4}+\frac{1}{4}}\right] = \log e^{\frac{1}{2}}$$

$$= \frac{1}{2}\log e, \quad \text{but } \log e = 1$$

$$= \frac{1}{2}\times 1 = \frac{1}{2}$$

$$\therefore f(0) = \frac{1}{2}$$ *(1 mark)*

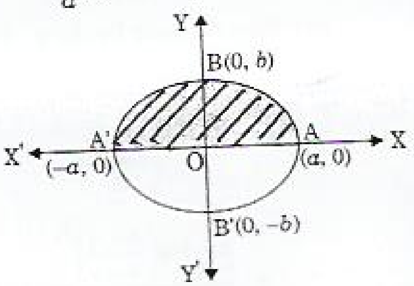
(iii) Find the volume of solid generated by revolving the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ about the major axis. (4)

Sol.: Consider X-axis as the major axis.

Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$.

$$\Rightarrow \frac{y^2}{b^2} = 1-\frac{x^2}{a^2}$$

$$\Rightarrow y^2 = \frac{b^2}{a^2}\left(a^2-x^2\right)$$



(1 mark)

Let V be the required volume of the solid obtained by revolving the ellipse about the major axis.

$$\therefore V = \pi\int_{-a}^{a} y^2\,dx$$

$$= \pi\int_{-a}^{a}\frac{b^2}{a^2}\left(a^2-x^2\right)dx$$ *(1 marks)*

$$= \frac{b^2}{a^2}\cdot\pi\cdot 2\int_0^a\left(a^2-x^2\right)dx$$

$$= \frac{2b^2}{a^2}\cdot\pi\left[a^2x-\frac{x^3}{3}\right]_0^a$$

$$= 2\pi\frac{b^2}{a^2}\left[a^2\cdot a-\frac{a^3}{3}\right]$$

$$= 2\pi\frac{b^2}{a^2}\left[a^3-\frac{a^3}{3}\right]$$ *(1 mark)*

$$= 2\pi\frac{b^2}{a^2}\left[\frac{3a^3-a^3}{3}\right] = \frac{2}{3}\pi\frac{b^2}{a^2}\left[2a^3\right]$$

$$\therefore V = \frac{4}{3}\pi ab^2 \text{ cubic units.}$$ *(1 mark)*

Section-II

Q.4. Attempt any SIX of the following : [12]

(i) What must be subtracted from each of the numbers 5, 7 and 10, so that the resulting numbers are in continued proportion ? (2)

Sol.: Let x be the number subtracting from each so that the numbers are in continued proportion.

i.e. $$\frac{5-x}{7-x} = \frac{7-x}{10-x}$$ *(1 mark)*

$$\Rightarrow (5-x)(10-x) = (7-x)^2$$

$$\Rightarrow 50-15x+x^2 = 49-14x+x^2$$

$$\Rightarrow 50-49 = 15x-14x$$

$$\Rightarrow 1 = x$$

$$\Rightarrow x = 1$$ *(1 mark)*

(ii) Amar, Akbar and Anthony started a transport business by investing ₹ 1 lakh each. Amar left after 5 months from the commencement of business and Akbar left 3 months later. At the end of one year, the business realised a profit of ₹ 37,500. Find the share of profit of each partner. (2)

Sol.: Given : Total profit for the year = ₹ 37,500

Proportion of period = 5 : 8 : 12

Let Amar's share of profit = ₹ $5x$

Akbar's share of profit = ₹ $8x$

Anthony's share of profit = ₹ $12x$

$\therefore 5x + 8x + 12x = 37,500$ *(1 mark)*

i.e. $25x = 37,500$

$\therefore x = \frac{37,500}{25} = 1,500$

$\therefore$ Amar's share of profit = 5 × 1500 = ₹ 7,500

Akbar's share of profit = 8 × 1500 = ₹ 12,000

Anthony's share of profit $= 12 \times 1500$

$= ₹ 18{,}000$

(1 mark)

(iii) At 7% rate of commission, Miss Mary, a sales girl, got ₹ 210 on the sales of toothpaste. Find the value of the sales if the price of toothpaste is ₹ 15 per container. Also find the number of containers sold the sales girl. (2)

Sol.: Since the rate of commission is 7% and Miss Mary received ₹ 210, total worth of her sales

$$= 210 \times \frac{100}{7} = 3{,}000$$

∴ Total value of the sales = ₹ 3,000 *(1 mark)*

The price of each container = ₹ 15

∴ Total number of containers sold

$$= \frac{3000}{15} = 200$$ *(1 mark)*

(iv) A book seller paid ₹ 765 for a bundle of 50 books on which he had been given a discount of 15%. Find the list price of a book. (2)

Sol.: Net price per book paid by the bookseller

$$= \frac{765}{50} = ₹\ 15.30$$ *(1 mark)*

The bookseller has been given 15% discount on the list price.

$$\therefore \text{List price of a book} = \frac{15.30 \times 100}{85}$$

$= ₹ 18$ *(1 mark)*

(v) A cottage is insured for 80% of its value. The annual premium at $\frac{7}{10}$% amounts to ₹ 280. Fire damaged the cottage to the extent of 40% of its value. Find how much can be claimed under the policy. (2)

Sol.: Insured value $= \dfrac{280 \times 100}{\left(\frac{7}{10}\right)} = 40{,}000$

$$\therefore \text{Property value} = \frac{40{,}000}{80} \times 100 = 50{,}000$$

$$\text{Loss} = ₹\ \frac{50{,}000 \times 40}{100} = ₹\ 20{,}000$$

$$\text{Claim} = \frac{\text{Loss} \times \text{Policy value}}{\text{Property value}}$$ *(1 mark)*

$$= \frac{20{,}000 \times 40{,}000}{50{,}000}$$

$= ₹ 16{,}000$ *(1 mark)*

(vi) From the following table find the sector which has more healthy population. (2)

Age Group	Age SDR		Standard Population
	Sector I	Sector II	
0 – 20	10	8	20,000
20 – 40	5	6	40,000
40 – 60	8	5	10,000

Sol.: If P_i is the population of a certain age-group and M_i is the Age SDR of the age-group, then using the given information we get

$$\sum P_i = 70{,}000$$

Age group	Age SDR of Sector - I (M_{1_i})	Age SDR of Sector - II (M_{2_i})	Standard Population (P_i)	$M_{1_i} \times P_i$	$M_{2_i} \times P_i$
0 – 20	10	8	20,000	2,00,000	1,60,000
20 – 40	5	6	40,000	2,00,000	2,40,000
40 – 60	8	5	10,000	80,000	50,000
Total			70,000	4,80,000	4,50,000

(1 mark)

From the above table, $\sum P_i = 70{,}000$

For Sector I, $\sum M_{1_i} \times P_i = 4{,}80{,}000$

For Sector II, $\sum M_{2_i} \times P_i = 4{,}50{,}000$

Now, STDR for Sector I

$$= \frac{\sum M_{1_i} \times P_i}{\sum P_i} = \frac{4{,}80{,}000}{70{,}000} = 6.86$$

STDR for Sector II

$$= \frac{\sum M_{2_i} \times P_i}{\sum P_i} = \frac{4{,}50{,}000}{70{,}000} = 6.4$$

$\because \quad (STDR)_I > (STDR)_{II}$

Hence population of Sector II is more healthy than population of Sector I. *(1 mark)*

(vii) For a group of 30 couples the regression line of the age of wife (y) on the age of husband (x) is given by $3y - 4x + 60 = 0$.

If $\bar{y} = 40$ and $\frac{\sigma_x^2}{\sigma_y^2} = \frac{9}{25}$, find $\bar{x}$ and r, where $\bar{x}$ and r have their usual meaning. (2)

Sol.: Given that : $n = 30, \quad \bar{y} = 40$

and $\quad \dfrac{\sigma_x^2}{\sigma_y^2} = \dfrac{9}{25} \Rightarrow \dfrac{\sigma_x}{\sigma_y} = \dfrac{3}{5}$

Equation of regression line of Y on X is

$3y - 4x + 60 = 0$

$\therefore\ 3y = 4x - 60$

$\therefore\ y = \frac{4}{3}x - 20 \quad ....(i)$

$\therefore\ b_{yx} = \text{coefficient of } x = \frac{4}{3}$

But (i) passes through $(\bar{x}, \bar{y})$, i.e. $(\bar{x}, 40)$

$\therefore\ \bar{y} = \frac{4}{3}\bar{x} - 20$

$\Rightarrow 40 = \frac{4}{3}\bar{x} - 20 \quad \Rightarrow 40 + 20 = \frac{4\bar{x}}{3}$

$\Rightarrow \frac{60 \times 3}{4} = \bar{x}$

$\therefore\ \bar{x} = 45$ *(1 mark)*

Also, $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

$\Rightarrow \frac{4}{3} = r \times \frac{5}{3} \ ...\left(\because \frac{\sigma_x}{\sigma_y} = \frac{3}{5},\ \therefore \frac{\sigma_y}{\sigma_x} = \frac{5}{3}\right)$

$\Rightarrow r = \frac{4}{5}$ *(1 mark)*

(viii) If X has Poisson distribution with variance 2, find $P(X \le 4)$. [Use $e^{-2} = 0.1353$]

Also find the mean value of X. **(2)**

Sol.: Given : $X \sim P(m)$

$\therefore$ p.d.f. $= P(X = x) = \frac{e^{-m} \cdot m^x}{x!}$

For a Poisson distribution,

Mean = Variance = 2 (given)

i.e. $m = 2$ $\therefore$ Mean = 2

$\therefore P(X = x) = \frac{e^{-2} \cdot 2^x}{x!}$ *(1 mark)*

$\therefore$ Required Probability

$= P(X = 4) = \frac{e^{-2} \cdot 2^4}{4!}$

[Given : $e^{-2} = 0.1353$]

$= \frac{0.1353 \times 16}{1 \times 2 \times 3 \times 4}$

$= 0.0902$ *(1 mark)*

Q.5.(A) Attempt any TWO of the following : [6]

(i) Given : $u = x - 15$, $v = y - 65$, $n = 10$ $\sum u = 10$, $\sum v = 10$, $\sum uv = 94$, $\sum u^2 = 90$, $\sum v^2 = 208$, find the coefficient of correlation, r. **(3)**

Sol.: $r = \frac{\sum uv - \frac{\sum u \cdot \sum v}{n}}{\sqrt{\sum u^2 - \frac{(\sum u)^2}{n}} \cdot \sqrt{\sum v^2 - \frac{(\sum v)^2}{n}}}$

(1 mark)

$= \frac{94 - \left(\frac{10 \times 10}{10}\right)}{\sqrt{90 - \frac{100}{10}} \cdot \sqrt{208 - \frac{100}{10}}}$

$= \frac{94 - 10}{\sqrt{80} \times \sqrt{198}}$

$= \frac{84}{125.86}$

$= 0.67$ *(1 mark)*

(ii) If a random variable X has Binomial variate with $E(X) = 4$ and standard deviation of $X = \sqrt{3}$. Find the values of n, p and q. **(3)**

Sol.: Given : $X \sim B(n, p)$

and $E(X)$ = mean $= np = 4$

Also, given standard deviation $= \sqrt{3}$

Variance $= (S.D.)^2 = 3$

$\therefore npq = 3$, but $np = 4$

$\Rightarrow 4q = 3 \quad \Rightarrow q = \frac{3}{4}$ *(1 mark)*

$p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore p = \frac{1}{4}$ *(1 mark)*

Since $np = 4$

$\Rightarrow n \times \frac{1}{4} = 4 \quad \Rightarrow n = 16$ *(1 mark)*

$\therefore n = 16, \quad p = \frac{1}{4}, \quad q = \frac{3}{4}.$

(iii) Calculate the crude death rate (CDR) for the following data :

Age group	Population	Deaths
20 – 25	12,000	372
25 – 30	30,000	660
30 – 35	62,000	1612
35 – 40	15,000	525
40 – 50	3,000	180

(3)

Sol.: $\text{CDR} = \frac{\sum D}{\sum P} \times 1000$ *(1 mark)*

$\sum D = 372 + 660 + 1612 + 525 + 180$

$= 3,349$

$\sum P = 12,000 + 30,000 + 62,000 + 15,000 + 3,000$

$= 1,22,000$ *(1 mark)*

$\therefore \text{CDR} = \frac{3349}{122000} \times 1000$

$= 27.45$ per thousand *(1 mark)*

Q.5.(B) Attempt any TWO of the following : [8]

(i) The equations of two regression lines are $10x+3y-62=0$ and $6x+5y-50=0$, find
(a) b_{yx} (b) b_{xy} (c) σ_x, if $\sigma_y = 2$ (d) r. (4)

Sol.: Let equation of line of regression of x on y be

$10x+3y-62 = 0$

$\Rightarrow 10x = -3y+62$

$\Rightarrow x = -\frac{3}{10}y+\frac{62}{10}$

$\therefore b_{xy} = \text{Coefficient of } y = -\frac{3}{10}$ *(1 mark)*

Let equation of line of regression of x on y be

$6x+5y-50 = 0$

$\Rightarrow 5y = -6x+50$

$\Rightarrow y = -\frac{6}{5}x+\frac{50}{5}$

$\therefore b_{yx} = \text{Coefficient of } x = -\frac{6}{5}$ *(1 mark)*

$\therefore b_{xy} = -\frac{3}{10}$ and $b_{yx} = -\frac{6}{5}$

Now, $r = \pm\sqrt{b_{yx}\times b_{xy}} = \pm\sqrt{\frac{-3}{10}\times\frac{-6}{5}}$

$= \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$

Since both b_{xy} and b_{yx} are negative, r is also negative.

$\therefore r = -\frac{3}{5} = -0.6$ *(1 mark)*

Given : $\sigma_y = 2$ and we have to find σ_x.

Since $b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$\Rightarrow -\frac{6}{5} = -\frac{3}{5}\times\frac{2}{\sigma_x}$ $\Rightarrow 2 = \frac{2}{\sigma_x}$

$\Rightarrow \sigma_x = 1$

$\therefore \sigma_x = 1$ *(1 mark)*

(ii) Solve the following assignment problem so as to maximize the total profit.

		Sale of Products		
		A	**B**	**C**
Outlets	**I**	**30**	**40**	**50**
	II	**80**	**80**	**150**
	III	**40**	**60**	**80**

(4)

Sol.: As the problem is of maximization, subtracting each element from 150, we get

	A	B	C
I	120	110	100
II	70	70	0
III	110	90	70

(1 mark)

Subtracting the smallest element of each row and column from the respective row and column elements, we get

	A	B	C
I	20	10	0
II	70	70	0
III	40	20	0

→

	A	B	C
I	0	0	0
II	50	60	0
III	20	10	0

(1 mark)

Only two lines are sufficient to cover all zeros. Hence, subtracting smallest of the uncovered from the uncovered elements, we get

	A	B	C
I	$\boxed{0}$	$\not{0}$	$\not{0}$
II	40	40	$\boxed{0}$
III	10	$\boxed{0}$	$\not{0}$

(1 mark)

Allocating row-wise and column-wise we get optimum solutions.

$I \to A, II \to C$ and $III \to B$.

$\therefore$ Total maximum profit $= 30 + 150 + 60$

$= 240$ *(1 mark)*

(iii) An aeroplane can carry a maximum of 200 passengers. Baggage allowed for a first class passenger is 30 kgs and for an economy class passenger is 20 kgs. The maximum capacity of the aeroplane for the baggage is 4500 kgs. The profit on each first class ticket is ₹ 500 and that on economic class ticket is ₹ 300. It is required to determine how many tickets of each type be sold so as to get maximum profit. Formulate this problem as L.P.P. and solve it graphically and write the vertices of feasible region. (4)

Sol.: Let x tickets of first class and y tickets of economy class be sold.

$x \ge 0,\ y \ge 0$ ....(i)

As the aeroplane can carry maximum 200 passengers,

$x + y \le 200$ ....(ii)

Therefore we can tabulate the given data as:

	First Class Ticket	Economy Class Ticket	Maximum Capacity
Baggage (in kgs.)	30	20	4500
Profit (in ₹)	500	300	—

$\therefore 30x + 20y \le 4500$ ....(iii)

Let Z denote total profit.

$\therefore$ Z = $500x + 300y$ ...(objective function) *(1 mark)*

$\therefore$ L.P.P. is to Maximize Z = $500x + 300y$

subject to $x + y \le 200$

$30x + 20y \le 4500$

$x \ge 0,\ y \ge 0$ *(1 mark)*

We plot the lines $x + y = 200$ ....(iv)

and $30x + 20y = 4500$

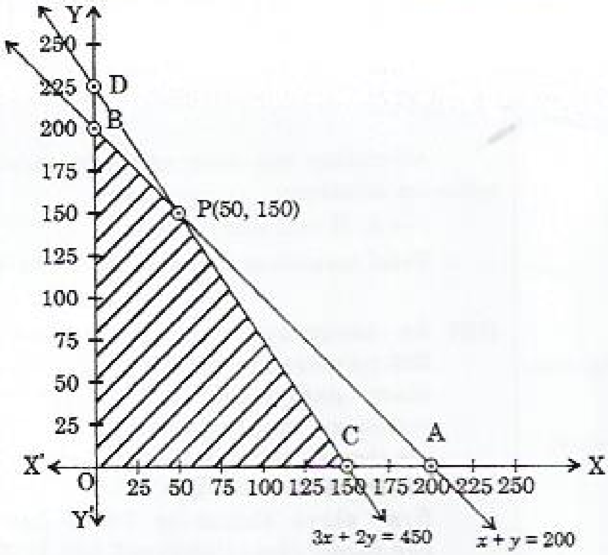
i.e. $3x + 2y = 450$ ....(v)

$x + y = 200$ ....(vi)

and consider $3x + 2y = 450$.

x	0	150
y	225	0

x	0	200
y	200	0



(1 mark)

The feasible region is the quadrilateral OCPB, where P is the point of intersection of the lines $x + y = 200$ and $3x + 2y = 450$.

Solving equations (v) and (vi) we get,

$x = 50$ and $y = 150$.

$\therefore$ P $\equiv (50, 150)$.

$\therefore$ Vertices of feasible regions are

(0, 0), (150, 0), (50, 150) and (0, 200) *(1 mark)*

Q.6.(A) Attempt any TWO of the following : [6]

(i) If for the following data the C.D.R. is 55, find the value of x.

Age group	Population in thousands	Number of deaths
0 – 25	25	1250
25 – 40	x	1000
40 – 70	28	1570
70 and above	15	1680

(3)

Sol.: $\text{C.D.R.} = \dfrac{\text{Total number of death}}{\text{Total population}} \times 1000$

i.e. $\text{C.D.R.} = \dfrac{D}{P}$, where *(1 mark)*

D = Total death

= 1250 + 1000 + 1570 + 1680

= 5500

P = Total population in thousands

$= 25 + x + 28 + 15$

$= 68 + x$

$\therefore 55 = \dfrac{5500}{68 + x}$ *(1 mark)*

$\Rightarrow 68 + x = 100$

$\therefore x = 100 - 68 = 32$

$\therefore x = 32$ *(1 mark)*

(ii) The following data gives the marks of 25 students in Mathematics (X) and Accountancy (Y) in an examination out of 5.

(5, 4), (3, 2), (4, 5), (5, 4), (5, 5), (2, 1), (5, 3), (3, 1), (1, 1), (2, 2), (3, 2), (3, 3), (5, 4), (5, 2), (4, 2), (5, 3), (4, 1), (2, 2), (2, 1), (3, 1), (4, 3), (5, 3), (5, 4), (4, 3), (3, 2).

Construct a bivariate frequency table and answer the following

(a) How many students get marks less than 4 in Mathematics ?

(b) How many students get marks more than 3 in Accountancy ?

(c) How many students have 100% marks in at least one paper ? (3)

Sol.: Bivariate Frequency Table

Y↓ \ X→	1	2	3	4	5	Total
1	| (1)	|| (2)	|| (2)	| (1)	—	6
2	—	|| (2)	||| (3)	| (1)	| (1)	7
3	—	—	| (1)	|| (2)	||| (3)	6
4	—	—	—	—	|||| (4)	4
5	—	—	—	| (1)	| (1)	2
Total	1	4	6	5	9	25

From the bivariate frequency table :

(a) Number of students who got marks less than 4 in Mathematics = 5 + 5 + 1 + 0 + 0

= 11 *(1 mark)*

(b) Number of students who got marks more than 3 in Accountancy = 4 + 1 + 1

= 6 *(1 mark)*

(3) Number of students who have 100% marks (i.e. 5 marks) in at least one paper

= 1 + 3 + 4 + 1 + 1 = 10 *(1 mark)*

(iii) Find k, if the function f defined by

$$f(x) = kx, \quad 0 < x < 2$$
$$= 0, \quad \text{otherwise}$$

is the p.d.f. of a random variable X.

Also find $P\left(\frac{1}{4} < x < \frac{1}{3}\right)$. **(3)**

Sol.: Since $f(x)$ is p.d.f., $\int_0^2 f(x)\,dx = 1$.

$$\therefore \int_0^2 (kx)\,dx = 1 \quad \Rightarrow k\left[\frac{x^2}{2}\right]_0^2 = 1$$

$$\Rightarrow k[2] = 1 \quad \Rightarrow k = \frac{1}{2}$$ *(1 mark)*

Thus, $f(x) = \frac{x}{2}, \quad 0 < x < 2$

Now, $P\left(\frac{1}{4} < X < \frac{1}{3}\right)$

$$= \int_{1/4}^{1/3} f(y)\,dx \quad = \int_{1/4}^{1/3} \frac{x}{2}\,dx$$ *(1 mark)*

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{1/4}^{1/3} \quad = \frac{1}{4}\left[x^2\right]_{1/4}^{1/3}$$

$$= \frac{1}{4}\left[\left(\frac{1}{3}\right)^2 - \left(\frac{1}{4}\right)^2\right] \quad = \frac{1}{4}\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= \frac{1}{4}\left[\frac{7}{144}\right] \quad = \frac{7}{576}$$ *(1 mark)*

Q.6.(B) Attempt any TWO of the following : **[8]**

(i) Find the rank correlation coefficient for the following data :

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

(4)

Sol.:

X	Y	Rank X (R_1)	Rank Y (R_2)	$d = R_2 - R_1$	d^2
68	62	4	5	1	1
64	58	6	7	1	1
75	68	2.5	3.5	1	1
50	45	9	10	1	1
64	81	6	1	−5	25
80	60	1	6	5	25
75	68	2.5	3.5	1	1
40	48	10	9	−1	1
55	50	8	8	0	0
64	70	6	2	−4	16
Total					72

(1 mark)

C.F. for 64, $m = 3$.

$$\therefore CF_1 = \frac{m(m^2-1)}{12} = \frac{3(3^2-1)}{12}$$
$$= \frac{3\times 8}{12} = 2$$

C.F. for 72, $m = 2$.

$$\therefore CF_2 = \frac{m(m^2-1)}{12} = \frac{2(2^2-1)}{12}$$
$$= \frac{2\times 3}{12} = 0.5$$

C.F. for 68, $m = 2$.

$$\therefore CF_3 = \frac{m(m^2-1)}{12} = \frac{2(2^2-1)}{12}$$
$$= \frac{2\times 3}{12} = 0.5$$

$$\text{Total C.F.} = CF_1 + CF_2 + CF_3$$
$$\therefore \quad = 2 + 0.5 + 0.5$$
$$= 3$$
$$\sum d^2 = 72$$ *(1 mark)*

$\therefore$ Rank Correlation Coefficient,

$$R = 1 - \frac{6\left[\sum d^2 + \text{C.F.}\right]}{n(n^2-1)}$$ *(1 mark)*

i.e. $$R = 1 - \frac{6[72+3]}{10(10^2-1)} = 1 - \frac{6\times 75}{10\times 99}$$

$$= 0.545$$ *(1 mark)*

(ii) The time required by each worker to complete each job is given below, where '—' means the particular job cannot be assigned to the particular worker. Find the assignment jobs to workers which will minimise the total time required to complete all the jobs.

Jobs	Workers		
	A	B	C
I	12	10	8
II	8	9	11
III	11	—	12

(4)

Sol.: We replace '—' be a very large number M so that adding or subtracting a small quantity does not affect M.

	A	B	C
I	12	10	8
II	8	9	11
III	11	M	12

Subtracting the smallest element of each row from all its elements, we get

	A	B	C
I	4	2	0
II	0	1	3
III	0	M	1

← Step I

(1 mark)

Subtracting the least element of each column from all its elements.

Draw minimum number of lines required to cover all zeros = 3, which is equal to the order of the matrix.

	A	B	C
I	4	1	0
II	0	0	3
III	0	M	1

← Step II

(1 mark)

∴ Optimal solution is reached.

We examine the rows for single zero and make an assignment. Then cancel any other zeros to the corresponding column.

	A	B	C
I	4	1	[0]
II	~~0~~	0	3
III	[0]	M	1

← Step III

Examine columns for a single zero, make an assignment and cancel the zeros of the row.

	A	B	C
I	4	1	[0]
II	~~0~~	[0]	3
III	[0]	M	1

← Step IV

(1 mark)

∴ The optimal assignment is

I → C, II → B, III → A *(1 mark)*

(iii) Four freelancers P, Q, R, S can do four types of jobs. The corresponding cost is given below. If each person is to be assigned exactly one job, solve the problem for minimising the cost.

Jobs	**Persons (Freelancers)**			
	P	**Q**	**R**	**S**
J_1	**0**	**18**	**9**	**3**
J_2	**10**	**25**	**1**	**23**
J_3	**24**	**5**	**4**	**1**
J_4	**9**	**16**	**14**	**0**

(4)

Sol.: Step I : We subtract least element of each row from all the elements.

	P	Q	R	S
J_1	0	18	9	3
J_2	9	24	0	22
J_3	23	4	3	0
J_4	9	16	14	0

(1 mark)

Step II : Subtracting least element of each column from all its elements, we get

	P	Q	R	S
J_1	0	14	9	3
J_2	9	20	0	22
J_3	23	0	3	0
J_4	9	12	14	0

(1 mark)

As each row and each column have at least one zero, we draw minimum horizontal/vertical lines to cover all zeros.

	P	Q	R	S
J_1	0	14	9	3
J_2	9	20	0	22
J_3	23	0	3	0
J_4	9	12	14	0

As (minimum number of lines)

= 4 = (Order of the matrix)

∴ Optimal solution is reached.

Step III : We examine rows for a single zero and make an assignment there.

Then cross any other zeros present in the corresponding columns.

	P	Q	R	S
J_1	[0]	14	9	3
J_2	9	20	[0]	22
J_3	23	[0]	3	~~0~~
J_4	9	12	14	[0]

(1 mark)

Step IV : We examine the columns where assignment is not made for single zero and make an assignment.

	P	Q	R	S
J_1	[0]	14	9	3
J_2	9	20	[0]	22
J_3	23	[0]	3	~~0~~
J_4	9	12	14	[0]

∴ The optimal assignment is

$J_1 \to P$, $J_2 \to R$, $J_3 \to Q$ and $J_4 \to S$ *(1 mark)*